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Landau levels in the presence of topological defects

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Abstract

In this paper we study the Landau levels in the presence of topological defects. We analyse the behaviour of electrons moving in a magnetic field in the presence of a continuous distribution of disclinations, a magnetic screw dislocation and a dispiration. We focus on the influence of these topological defects on the spectrum of the electron (or hole) in the magnetic field in the framework of the geometric Katanaev–Volovich theory of defects in solids. The presence of the defect breaks the degeneracy of the Landau levels in different ways depending on the defect. Exact expressions for energies and eigenfunctions are found for all cases. Using Kaluza–Klein theory we solve the Landau level problem for a dispiration and compare the results with the ones obtained in the previous cases.

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1. Introduction

The role played by topology in the physical properties of a variety of systems has been a very important issue in different areas of physics such as, for example, gravitation and condensed matter physics. Topological defects appear in gravitation as monopoles, strings and walls [1]. In condensed matter physics they are vortices in superconductors or superfluids [2], domain walls in magnetic materials [2], solitons in quasi-one-dimensional polymers [3] and dislocations or disclinations in disordered solids or liquid crystals [2, 4]. The change in the topology of a medium introduced by a linear defect such as a disclination, dislocation or dispiration in an elastic media or a cosmic defect in space-time produces some effects on the physical properties of the medium [5, 6].

In classical theory of elasticity, defects are described by the strain and stress tensors that contain all the information about the deformation caused by them in the continuous media. Distortions were described by Volterra [7] in the context of elasticity theory and later were the subject of investigations in the context of both solid continua and crystals [4, 8]. We consider as a prototype of such an object, a hollow cylinder made of elastic material and cut it at a half two-plane, e.g. at $\phi = 0$ (using cylindrical coordinates $\{r, \phi, z\}$, the cylinder is taken to be oriented along the z-axis), thereby destroying its multiple connectivity. Then take the two lips that have been separated by the cut and translate and rotate them against each other. Finally, after eventually removing superfluous or adding missing material, weld the two planes together again. This cutting and welding process is called the Volterra process.

The influence of line defects on the electronic properties of a crystal is an old issue in condensed matter physics [9]. Recently, alternative approaches to study this problem have been proposed which use either a gauge field [5, 10] or a gravity-like approach [2, 11–13]. The modelling of the influence of the defects on the quantum motion of electrons becomes reasonably easy in the latter approach, where the boundary conditions imposed by the defect are incorporated into a metric. This geometric approach is based on the isomorphism that exists between the theory of defects in solids and three-dimensional gravity [14]. From the mathematical point of view, geometric theory of defects in solids and the theory of gravity with torsion in the Euclidean formulation are the same models. This approach is the Katanaev–Volovich theory of defects in solids. The advantage of this geometric description of defects in solids is twofold. Firstly, in contrast to the ordinary elasticity theory this approach provides an adequate language for continuous distribution of defects. Secondly, a mighty mathematical machinery of differential geometry clarifies and simplifies calculations. In recent years alternative approaches for geometric descriptions of defects have been presented by several authors [15–18].

In recent years, the influence of geometry in the Landau levels has been the object of intensive research. Dunne [19] and Comtet [20] analyse the influence of the hyperbolic and spherical geometries in the Landau levels. The influence of topology in Landau levels was also investigated recently [21, 22].

In this paper we investigate the influence of topology in the study of Landau levels in the presence of topological defects in condensed matter physics, studying the effects of different topological defects (density of disclinations, dislocation and dispiration) on the energy spectrum of a point charge (electrons or holes) in the presence of a magnetic field parallel to the defect. The only approximation we make is to work in the continuum elastic medium, where the geometric approach makes sense [23, 24]. In general, defects correspond to a singular curvature or torsion (or both) along the defect line [14]. The treatment that we adopt to determine Landau levels in a medium with defects is the following: defects are described by a metric g_{ij} that contains all information about the deformation caused in the medium by them; in each case the metric is a solution of Einstein equation for three-dimensional media with curvature and/or torsion.

The Hamiltonian corresponding to a charged particle in the presence of a vector potential in the background given by metric g_{ij} is

$$H = \frac{1}{2m\sqrt{g}}\left(p_i - \frac{q}{c}A_i\right)\left[\sqrt{g}g^{ij}\left(p_j - \frac{q}{c}A_j\right)\right] \tag{1}$$

where the minimal coupling $\vec{p} \rightarrow \vec{p} - q/c\vec{A}$ was used and q is the electrical charge of the particle. In the literature the study of Landau levels is performed by use of minimal coupling of the linear momentum and potential vector, $\vec{p} \rightarrow \vec{p} - \frac{e}{c}\vec{A}$. In this paper we introduce a new approach that consists in the use of Kaluza–Klein [25, 26] theory to obtain, from the geometrical point of view, the Landau levels, introducing the magnetic field in the metric that describes the defect.

This paper is organized as follows: in sections 2, 3 and 4 we investigate the Landau levels in the presence of a density of disclinations, a magnetic screw dislocation and a dispiration, respectively. In section 5 we study the Landau levels in the context of Kaluza–Klein theory and consider the solution that corresponds to a dispiration in this framework, and finally, in section 6, we present the concluding remarks.

2. Landau levels in the presence of a continuous distribution of disclinations

In a recent paper [22] we have discussed the quantum motion of electrons (or holes) under a uniform magnetic field, in the presence of a disclination. In the geometric approach, the medium with a disclination has the line element given by

$$\mathrm{d}s^2 = g_{ij}\,\mathrm{d}x^i\,\mathrm{d}y^j = \mathrm{d}z^2 + \mathrm{d}\rho^2 + \alpha^2\rho^2\,\mathrm{d}\phi^2 \tag{2}$$

in cylindrical coordinates. This metric is equivalent to the boundary condition with periodicity of $2\pi\alpha$ instead of 2π around the z-axis. In the Volterra process [4] of disclination creation, this corresponds to remove ($\alpha < 1$) or insert ($\alpha > 1$) a wedge of material of dihedral angle $\lambda = 2\pi(\alpha - 1)$. This metric corresponds to a locally flat medium with a conical singularity at the origin. The only nonzero components of the Riemann curvature tensor and the Ricci tensor are given by

$$R_{12}^{12} = R_1^2 = R_2^2 = 2\pi\frac{1-\alpha}{\alpha}\delta_2(\rho) \tag{3}$$

where $\delta_2(\rho)$ is the two-dimensional delta function in flat space. From the expression above, it follows that if $0 < \alpha < 1(-2\pi < \lambda < 0)$ the defect carries positive curvature and if $1 < \alpha < \infty(0 < \lambda < \infty)$ the defect carries negative curvature. This fact is very important in the curved space theory of amorphous solids [27] where geometrical frustration, the incompatibility between a given local order and the geometry of Euclidean space, is relieved by propagation of the local order in a space of constant curvature. Disclinations carrying curvature of sign opposite that of the curvature of the background space must be introduced [28] in order to reduce the mean curvature of the model to zero, yielding a distorted(locally curved) structure with the desired local order, perforated by disclination lines: a sensible structural model for amorphous solids.

The study of the Landau levels in the presence of a single disclination was already performed [22]. In this section we perform the calculation for a continuous distribution of disclinations. The importance of the study of this problem in a media with a distribution of defects is that in a real material, generally, one has a bigger number of defects instead of a single one.

In a recent paper, Katanaev and Volovich [29] considered a distribution of disclinations and obtained the metric which describes the continuous distribution of disclinations in an elastic media. We consider a circularly symmetric distribution of disclinations and assume that they are uniformly distributed on a disc of radius R with the following density of deficit angles:

$$\zeta = \begin{cases} q & \rho < R \\ 0 & \rho > R. \end{cases} \tag{4}$$

The normalized total deficit angle for this distribution is given by

$$\Theta = \tfrac{1}{2}qR^2. \tag{5}$$

Solving three-dimensional Einstein equation, the metric that describes the space outside the distribution of disclinations has the form

$$ds^2 = dz^2 + \rho^{qR^2}(d\rho^2 + \rho^2\, d\phi^2). \tag{6}$$

Now, performing the following change of coordinates:

$$\bar{\rho} = \frac{\rho^\alpha}{\alpha} \tag{7}$$

where

$$\alpha = 1 + \frac{qR^2}{2} \tag{8}$$

we obtain the metric for the density of disclinations, which is given by

$$ds^2 = dz^2 + d\bar{\rho}^2 + \alpha^2\bar{\rho}^2\, d\phi^2. \tag{9}$$

Therefore, performing an identification between the new parameter of density of disclinations and the old one used to describe a single disclination, we obtain that the line element for the density of disclinations has the same form of that corresponding to a single disclination, which means that the distribution of disclinations as seen from outside is the same as for one disclination with the deficit Θ.

In order to determine the Landau levels let us consider the covariant Schrödinger equation

$$\frac{1}{2m}\nabla^2\Psi = \mathrm{i}\frac{\partial\Psi}{\partial t} \tag{10}$$

where the Laplacian operator is

$$\nabla^2 = \frac{1}{\sqrt{g}}\partial_i\left(g^{ij}\sqrt{g}\partial_j\right) \tag{11}$$

and $g = \det|g_{ij}|$ stands for the determinant of the metric g_{ij}.

Therefore, the Schrödinger equation written in the space endowed by this metric, incorporates the boundary conditions dictated by the defect.

Now, let us determine the Landau levels in the presence of a distribution of disclinations. The configuration of the vector potential that gives a uniform magnetic field in conical space is

$$A(\rho) = \frac{B\rho}{2\alpha}\hat{e}_\phi. \tag{12}$$

This expression is very similar to the Euclidean case. The unique difference is the presence of the α factor in (12). By using minimal coupling $p_i \longrightarrow p_i - \frac{e}{c}\vec{A}$, we write the Hamiltonian for a particle in the presence of a distribution of disclinations submitted to a uniform magnetic field in the z-direction, that is given by

$$\hat{H} = -\frac{\hbar^2}{2m}\left\{\frac{\partial^2}{\partial z^2} + \frac{1}{\rho}\left(\rho\frac{\partial}{\partial\rho}\right) + \frac{1}{\alpha^2\rho^2}\frac{\partial^2}{\partial\phi^2}\right\} + \frac{\mathrm{i}\hbar\, q\, B}{2\alpha^2 m^* c}\frac{\partial}{\partial\phi} + \frac{q^2B^2}{8m\, c^2\alpha^2} \tag{13}$$

where we have dropped the bar in variable ρ.

In the limit $\alpha \longrightarrow 1$, expression (13) is the Euclidean Landau Hamiltonian. The solution of the Schrödinger equation corresponding to Hamiltonian (13) can be written in the form

$$\psi(z,\rho,\phi) = R(\rho)Z(z)\Phi(\phi) \tag{14}$$

where $Z(z) = \mathrm{e}^{\mathrm{i}kz}$ and $\Phi(\phi) = \mathrm{e}^{\mathrm{i}l\phi}$. Then, the solution of the radial equation is

$$R_{nl} = C_{nl}\exp\left(-\frac{|q|B\rho^2}{4c\hbar\alpha}\right)\rho^{\frac{|l|}{\alpha}}F\left(-n, \frac{|l|}{\alpha} + 1, \frac{|q|B\rho^2}{2c\hbar\alpha^2}\right) \tag{15}$$

where $F\left(-n, \frac{|l|}{\alpha}+1, \frac{|q|B\rho^2}{2c\hbar\alpha}\right)$ is the hypergeometric function, and C_{nl} is the normalization constant.

The energy levels for this case are given by

$$E = \frac{\hbar\omega_H}{2\alpha}\left(2n + \frac{|\ell|}{\alpha} \pm \frac{\ell}{\alpha} + 1\right) + \frac{\hbar^2 k^2}{2m^*} \tag{16}$$

with $n = 0, 1, 2, 3$ and $\ell = 0, \pm 1, \pm 2$, where the plus sign stands for holes ($q = |q|$), the minus sign for electrons ($q = -|q|$) and $\omega_H = \frac{|q|B}{m^*c}$ is the cyclotron frequency. Note that the effect of substituting an electron by a hole, $q \to -q$, is equivalent to inverting the sign of the z-component of the angular momentum, $\ell \to -\ell$. Since $\ell \notin Z$, electrons and holes have the same energy spectrum, the same z and radial wavefunctions and opposite cyclotron motions. In the limit $\alpha \to 1$, equation (16) gives the usual Landau eigenvalues plus the kinetic energy corresponding to the free motion along the z-axis. Note that when $\frac{1}{\alpha} = p$ is not an integer, the conicity introduced by the defect breaks the infinite degeneracy of the Landau levels.

3. Landau levels in the presence of a magnetic screw dislocation

Dislocations are much more realistic line defects. They can modify the energy spectrum of electrons moving in a uniform magnetic field as reported, for example, by Kaner and Feldman [23] and Kosevich [24]. In these earlier works, Landau levels in the presence of dislocations were investigated in the context of the classical theory of elasticity, via a perturbation in the Hamiltonian. The authors have considered the presence of a screw dislocation as a delta perturbation in the Hamiltonian of Landau levels.

We consider an infinitely long linear screw dislocation oriented along the z-axis. The three-dimensional geometry of the medium, in this case, is characterized by a nontrivial torsion which is identified with the surface density of the Burgers vector in the classical theory of elasticity. In this way, the Burgers vector can be viewed as flux of torsion. The screw dislocation is described by the following metric, in cylindrical coordinates [30]:

$$ds^2 = g_{ij}\,dx^i\,dy^j = (dz + \beta\,d\phi)^2 + d\rho^2 + \rho^2\,d\phi^2 \tag{17}$$

where β is a parameter related to the Burgers vector b by $\beta = \frac{b}{2\pi}$. This topological defect carries torsion but no curvature. The torsion associated with this defect corresponds to a conical singularity at the origin. The only nonzero component of the torsion tensor in this case is given by the two-form

$$T^1 = 2\pi\beta\delta^2(\rho)\,d\rho \wedge d\phi \tag{18}$$

where $\delta^2(\rho)$ is the two-dimensional delta function in flat space.

We are interested in the influence of the combined effect of the internal flux and the topological structure of the medium on the Landau levels. Therefore, let us assume the following idealized situation in which the topological defect carries in the core a magnetic field with magnetic flux given by Φ and outside the defect this magnetic field vanishes. This defect carries an internal magnetic flux and we call it a magnetic screw dislocation. We will consider in our calculations an internal magnetic field and an external uniform magnetic field and analyse the consequence of these fields in the Landau levels problem, especially in which concerns the internal magnetic field. The internal magnetic field is associated with a vector potential given by

$$A_{i,\phi} = \frac{\Phi}{2\pi\rho} \tag{19}$$

where Φ is the magnetic flux.

Next, we turn to the calculation of the vector potential for the external magnetic field in the space of a dislocation described by metric (17). We solve Maxwell equations $\vec{\nabla} \cdot \vec{B} = 0$ and $\vec{\nabla} \times \vec{B} = 4\pi \vec{J}$ considering the current density $\vec{J} = J_0 \delta(r - R)\hat{\phi}$, due to an infinite solenoid concentric with the dislocation axis. This ensures a uniform magnetic field $\vec{B}$ in the limit of infinite solenoid radius ($R \to \infty$). As a result we find $A_\phi(\rho) = \frac{B\rho}{2}$ for the non-zero component of the vector potential $\vec{A}$. Note that, in the non-Euclidean metric of the dislocation, the vector potential that produces the uniform magnetic field is identical to the flat space potential vector. The difference will appear in the differential operators, which must be defined according to the metric (17).

For a quasi-particle with effective mass m in the metric of the magnetic screw dislocation, the Schrödinger equation is

$$\left\{ -\frac{\hbar^2}{2m}\left[\partial_z^2 + \frac{1}{\rho}\partial\rho(\rho\partial_\rho) + \frac{1}{\rho^2}\left(\partial_\phi - \beta\partial_z + \frac{\Phi}{2\pi}\right)^2\right] + \frac{\mathrm{i}q\, B\,\hbar}{2mc}\left(\partial_\phi - \beta\partial_z + \frac{\Phi}{2\pi}\right) + \frac{q^2 B^2 \rho^2}{8m\, c^2}\right\}\varphi = E\psi. \tag{20}$$

This equation is solved using the ansatz

$$\psi(\phi, \rho, z) = C\mathrm{e}^{\mathrm{i}l\phi}\mathrm{e}^{\mathrm{i}kz}R(\rho) \tag{21}$$

where C is a normalization constant. Substituting this form of ψ into the Schrödinger equation we obtain the following radial equation:

$$\frac{1}{\rho}\partial_\rho(\rho\partial_\rho R(\rho)) - \frac{1}{\rho^2}\left(\ell - \beta k + \frac{\Phi}{2\pi}\right)^2 R(\rho) - \frac{q^2B^2\rho^2}{4\hbar^2c^2}R(\rho) - \frac{qB}{2\hbar c}\left(\ell - \beta k + \frac{\Phi}{2\pi}\right)R(\rho) - k^2R(\rho) + \varepsilon R(\rho) = 0 \tag{22}$$

with $\varepsilon = \frac{2mE}{\hbar^2}$. Now, using the change of variables $\sigma = \frac{\rho^2}{2}$, equation (22) is transformed into

$$\sigma^2\frac{\mathrm{d}^2R}{\mathrm{d}\sigma^2} + \sigma\frac{\mathrm{d}R}{\mathrm{d}\sigma} + \left\{\frac{A}{2}\sigma - \frac{q^2B^2\sigma^2}{4\hbar^2c^2} - \frac{(\ell - \beta + \frac{\Phi}{2\pi})^2}{4}\right\}R(\sigma) = 0 \tag{23}$$

where $A = \varepsilon - k^2 - \frac{qB}{2\hbar c}(\ell - \beta k + \frac{\Phi}{2\pi})$. The solution of this equation is given in terms of the confluent hypergeometric function F as

$$R(\rho) = C\exp\left\{-\frac{m\omega_B}{4\hbar}\rho^2\right\}\rho^{|\ell-\beta k+\frac{\Phi}{2\pi}|}F\left(-n, \left|\ell - \beta k + \frac{\Phi}{2\pi}\right| + 1, \frac{m\omega_B}{2\hbar}\rho^2\right) \tag{24}$$

where $\omega_B = \frac{qB_o}{mc}$. Then, the energy is given by

$$E = \hbar\omega_B\left\{n + \frac{|\ell - \beta k + \frac{\Phi}{2\pi}|}{2} - \frac{(\ell - \beta k + \frac{\Phi}{2\pi})}{2} + \frac{1}{2}\right\} + \frac{\hbar^2k^2}{2m} \tag{25}$$

with $n = 0, 1, 2, \ldots$, and the eigenfunction by

$$\psi(\rho, \phi, z) = C\,\mathrm{e}^{\mathrm{i}kz}\mathrm{e}^{\mathrm{i}\ell\phi}\exp\left\{\frac{m\omega_B\rho^2}{4\hbar}\right\}\rho^{|\ell-\beta k+\frac{\Phi}{2\pi}|}F\left(-n, \left|\ell - \beta k + \frac{\Phi}{2\pi}\right| + 1, \frac{m\omega_B}{2\hbar}\rho^2\right). \tag{26}$$

Note that, the energy levels have the infinite degeneracy of classical Landau levels broken by coupling the torsion β and the internal magnetic flux Φ of the defect with the angular momentum ℓ. In this case, the degeneracy of the levels is more strongly broken than in the case of a disclination. In the case of Landau levels, the coupling of angular momentum with parameter $\frac{1}{\alpha}$ is multiplicative and in the present case the coupling with parameter β is additive. This fact is responsible for the strong break of degeneracy of Landau levels in the presence of a screw dislocation. A special case will be considered if we observe that for some values of the magnetic field the influence of elastic properties of the medium can be cancelled by an internal magnetic field such that

$$\Phi = 2\pi\beta k. \tag{27}$$

4. Landau levels in the presence of a dispiration

In this section we analyse the Landau levels problem in the presence of dispiration. In general, this defect corresponds to singular curvature and torsion along the defect line [14]. We consider an infinitely long linear dispiration oriented along the z-axis. The three-dimensional geometry of the medium is characterized by nontrivial torsion and curvature which are identified with the surface density of the Burgers and Frank vectors, respectively, in the classical theory of elasticity. In this way, the Burgers vectors can be viewed as a flux of torsion and the Frank vector as a flux of curvature. This defect is described by the following metric:

$$\mathrm{d}s^2 = (\mathrm{d}z + \beta\,\mathrm{d}\phi)^2 + \alpha^2\rho^2\,\mathrm{d}\phi^2 + \mathrm{d}\rho^2 \tag{28}$$

where $\beta = \frac{b}{2\pi}$ and $\alpha = (1 + \frac{\lambda}{2\pi})$. This metric is equivalent to the following construction: removal ($\alpha < 1$) or insertion ($\alpha > 1$) of a wedge of material of dihedral angle $\lambda = 2\pi(\alpha - 1)$ followed by a translation of the lips with respect to each other of b along the z-direction, according to the Volterra process [4], b being the Burgers vector. The torsion two-form is the same as that of a screw dislocation and the curvature tensor the same of a disclination. This defect has two conical singularities, one due to torsion and the other to curvature. The vector potential that produces the uniform magnetic field in this metric is given by

$$A(\rho) = \frac{B\rho}{2\alpha}. \tag{29}$$

For a quasi-particle of effective mass m in the metric of the dispiration, the Hamiltonian is

$$\hat{H} = -\frac{\hbar^2}{2}\left\{\partial_z^2 + \frac{1}{\rho}\partial_\rho(\rho\partial_\rho) + \frac{1}{\alpha^2\rho^2}(\partial_\phi - \beta\partial_z)^2\right\} + \frac{\mathrm{i}qB\hbar}{2mc\alpha}(\partial_\phi - \beta\partial_z) + \frac{q^2B^2\rho^2}{8mc^2\alpha^2}. \tag{30}$$

The Schrödinger equation corresponding to this case can be solved using the ansatz

$$\psi(\phi, \rho, z) = C\mathrm{e}^{\mathrm{i}l\phi}\mathrm{e}^{\mathrm{i}kz}R(\rho) \tag{31}$$

where C is a normalization constant. Substituting this wave function into the Schrödinger equation we obtain the following radial equation:

$$\frac{1}{\rho}\partial_\rho(\rho\partial_\rho R(\rho)) - \frac{1}{\rho^2\alpha^2}(\ell - \beta k)^2 R(\rho) - \frac{q^2B^2\rho^2}{4\hbar^2c^2\alpha^2}R(\rho)$$
$$-\frac{qB}{2\hbar c\alpha}(\ell - \beta k)R(\rho) - k^2R(\rho) + \varepsilon R(\rho) = 0 \tag{32}$$

with $\varepsilon = \frac{2mE}{\hbar^2}$. Now, by using the change of variables $\sigma = \frac{\rho^2}{2}$, equation (32) is transformed into

$$\sigma^2\frac{\mathrm{d}^2R}{\mathrm{d}\sigma^2} + \sigma\frac{\mathrm{d}R}{\mathrm{d}\sigma} + \left\{\frac{A}{2}\sigma - \frac{q^2B^2\sigma^2}{4\hbar^2c^2} - \frac{(\ell - \beta)^2}{4}\right\}R(\sigma) = 0 \tag{33}$$

where $A = \varepsilon - k^2 - \frac{qB}{2\hbar c\alpha}(\ell - \beta k)$. The solution of this equation is given in terms of the confluent hypergeometric function F:

$$R(\rho) = C \exp\left\{-\frac{m\omega_B}{4\hbar\alpha}\rho^2\right\} \rho^{\frac{|\ell-\beta k|}{\alpha}} F\left(-n, \frac{|\ell - \beta k|}{\alpha} + 1, \frac{m\omega_B}{2\hbar\alpha^2}\rho^2\right) \tag{34}$$

where $\omega_B = \frac{qB_o}{mc}$. Then, the energy is

$$E = \frac{\hbar w_B}{\alpha}\left\{n + \frac{|l - \beta k|}{2\alpha} - \frac{(l - \beta k)}{2\alpha} + \frac{1}{2}\right\} + \frac{\hbar k^2}{2m} \tag{35}$$

where $n = 0, 1, 2, \ldots$, and the eigenfunction is given by the following expression:

$$\psi(\rho, \phi, z) = C e^{ikz} e^{i\ell\phi} \exp\left[\frac{m\omega_B \rho^2}{4\hbar\alpha}\right] \rho^{\frac{|l-\beta k|}{\alpha}} F\left(-n, \frac{|l - \beta k|}{\alpha} + 1, \frac{m\omega_B}{2\hbar\alpha^2}\rho^2\right). \tag{36}$$

From this expression we note that the presence of the defect breaks the degeneracy of the energy levels. Comparing this result with the one of the previous section, we conclude that in this case the degeneracy is strongly broken due to the influence of both parameters α and β. Note that if we take $\alpha = 0$, we get the results for a dislocation [21] and for $\beta = 0$, we get similar results for a disclination [22].

5. Landau levels in Kaluza–Klein theory

In contrast to the previous sections, where the magnetic field is introduced via minimal coupling, in this section the field is treated in the same way as the defects, that is, geometrically, in the framework of Kaluza–Klein theory. Then, let us consider the non-relativistic problem concerning Landau levels in the presence of a dispiration in this context. For this defect, the standard line element is given by

$$ds^2 = g_{\mu\nu}\, dx^\mu\, dx^\nu + g_{55}(dx - KA_\mu\, dx^\mu)^2 \tag{37}$$

where g_{55} is Kaluza–Klein scalar potential which will be considered as $g_{55} = 1$, K is the Kaluza constant and x is the fifth coordinate. In this context, the five-dimensional metric that corresponds to a uniform magnetic field in the presence of a dispiration is

$$ds^2 = d\rho^2 + (dz^2 + \beta\, d\phi)^2 + \alpha^2\rho^2\, d\phi^2 + \left(dx - \frac{B\rho^2}{2}\, d\phi\right)^2 \tag{38}$$

where $A_\phi = B\rho/(2\alpha)$ is the vector potential and the magnetic field is $B^z = B$. The metric contains the elastic deformation caused by the defect in the medium and has the coupling between the fifth coordinate and the ϕ coordinate.

Writing Schrödinger equation given by equation (10) in the space with metric (38), we get

$$-\frac{1}{2m}\partial_z^2 + \frac{1}{\rho}\partial_\rho(\rho\partial\rho) + \partial_x^2 + \frac{1}{\alpha^2\rho^2}\left(\partial_\phi - \beta\partial_z - \frac{B\rho^2}{2}\partial_x\right)^2 \psi = i\frac{\partial\psi}{\partial t}. \tag{39}$$

Now, let us consider the ansatz

$$\Psi(x, t, \rho, \phi, z) = \exp[-iEt + ikz + iQx + il\phi]R(\rho). \tag{40}$$

Substituting equation (40) into (39), we obtain the following radial equation:

$$\left\{\frac{1}{\rho}\frac{d}{d\rho} + \frac{1}{\alpha^2\rho^2}\left[\ell - \beta k - \frac{B\rho^2}{2}\right]^2 + (2mE - k^2 - Q^2)\right\} R(\rho) = 0. \tag{41}$$

In this case we adopt the same procedure that we have adopted in previous sections. Let us introduce a new variable η such that $\eta = \frac{\rho^2}{2}$. Then, equation (41) turns into

$$\eta^2 \frac{\mathrm{d}^2 R(\eta)}{\mathrm{d}\eta^2} + \eta \frac{\mathrm{d}R(\eta)}{\mathrm{d}\eta} - \frac{(\ell - \beta k)}{4\alpha^2} + \frac{BQ(\ell - \beta k)}{2\alpha^2} R(\eta) - \frac{B^2 Q^2 \eta^2}{4\alpha^2} R(\eta) + (A^2 - k^2 - Q^2) R(\eta) = 0 \qquad (42)$$

where $A^2 = 2mE$. The solution of this equation is the confluent hypergeometric function. Therefore, the wavefunction is given by

$$\Psi(t, \rho, \phi, z, x) = C_{n\ell} \mathrm{e}^{-\mathrm{i}Et + \mathrm{i}kz + \mathrm{i}Qx + \mathrm{i}\ell\phi} \mathrm{e}^{-\frac{BQ\rho^2}{4\alpha}} \rho^{|\ell - \beta k|} F\left(-n, \frac{|\ell - \beta k|}{\alpha} + 1, \frac{\rho^2}{2}\right) \qquad (43)$$

and the eigenvalues are

$$E = \frac{BQ}{m\alpha}\left(n + \frac{|\ell - \beta k|}{2\alpha} - \frac{\ell - \beta k}{2\alpha} + \frac{1}{2}\right) + \frac{K^2}{2m} + \frac{Q^2}{2m}. \qquad (44)$$

This result is in agreement with our earlier work [22] where the magnetic field was introduced by minimal coupling. Note that in the limit $\alpha \to 1$ we obtain the result of Landau levels in the presence of a screw dislocation. For $\beta = 0$, we obtain the results corresponding to a dislocation. If we define $\omega = \frac{BQ}{m}$, the eigenvalues (44) are almost the same of the dispiration, which were obtained using the minimal coupling. The unique difference is the extra term $\frac{Q^2}{2m}$ in equation (44), which is the kinetic energy associated with the fifth dimension, with quantum number given by the charge of the particle.

6. Concluding remarks

In this paper we study Landau levels in the presence of a class of topological defects. We demonstrate that the presence of topological defects breaks the infinite degeneracy of Landau levels, due to the unusual boundary conditions imposed by the defects. The advantage of this geometrical method to treat these problems are the easy and exact calculations employed, in contrast with the theory of elasticity, in which the exact solutions of the simple problem in general are not possible. Generally, the determination of Landau levels in the framework of the theory of elasticity are taken using perturbation methods to solve the Schrödinger equation [23, 24]. The degeneracy of the Landau levels in the presence of a continuous distribution of disclinations is smoothly broken. The coupling of the curvature of this conical defect to the angular momentum gives the possibility of breaking the degeneracy for some specific values of α. In the case of a screw dislocation, the coupling of Burgers vector $\beta = \frac{b}{2\pi}$ to the angular momentum ℓ is responsible for the breaking of degeneracy of Landau levels. In the case of a dislocation, we see that the torsion affects the Landau levels more strongly. For a dispiration, a defect that carries both curvature and torsion, we can conclude that the degeneracy of the Landau levels is more strongly broken than in all previous cases. The combination of the couplings of torsion β and curvature α is responsible for this effect on the energy levels. We use Kaluza–Klein theory as a unified geometric method to study Landau levels in the presence of defects. This approach permits us to treat the system formed by a defect plus a magnetic field in a purely geometrical way. In this theory, we introduce the elastic deformation of a media with the homogeneous magnetic field included in the metric that describes the dispirated media and we obtain the results concerning Landau levels in an elegant way.

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